Exercise 51

If g is a differentiable function, find an expression for the derivative of each of the following functions.

$$\text{(a)} \;\; y = xg(x) \qquad\qquad \text{(b)} \;\; y = \frac{x}{g(x)}$$

$$\text{(c)} \;\; y = \frac{g(x)}{x}$$

Solution

Use the product rule to differentiate the function in part (a).

$$y' = \frac{d}{dx}[xg(x)] = \left[\frac{d}{dx}(x)\right]g(x) + xg'(x) = (1)g(x) + xg'(x) = g(x) + xg'(x)$$

Use the quotient rule to differentiate the function in part (b).

$$y' = \frac{d}{dx}\left[\frac{x}{g(x)}\right] = \frac{\left[\frac{d}{dx}(x)\right]g(x) - g'(x)(x)}{[g(x)]^2} = \frac{(1)g(x) - g'(x)(x)}{[g(x)]^2} = \frac{g(x) - xg'(x)}{[g(x)]^2}$$

Use the quotient rule to differentiate the function in part (c).

$$y' = \frac{d}{dx}\left[\frac{g(x)}{x}\right] = \frac{g'(x)x - \left[\frac{d}{dx}(x)\right]g(x)}{x^2} = \frac{g'(x)x - (1)g(x)}{x^2} = \frac{xg'(x) - g(x)}{x^2}$$